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TRAVAUX SCIENTIFIQUES

FASCICULE N° 14

NOGENT-LE-ROTROU

IMPRIMERIE DAUPELEY-GOUVERNEUR

1936

P′

BY I. LEHMANN

I

P′ TIME-CURVES

1. The retardation observed in longitudinal waves at great epicentral distances is explained by the assumption of a decrease in their velocity in the interior of the earth. The course of the time-curve agrees roughly with a sudden fall in the velocity at a depth of about 2900 km.

For the purpose of illustrating the characteristics of the P and P' curves we have adopted the simple assumption of constant velocity of 10 km/sec in the mantle and of 8 km/sec in the core. The radius of the core is taken to be $\frac{5}{9}\, r_0$, $r_0 =$ 6370 km being the radius of the earth.

In Fig. 1 a ray passing only through the mantle has been drawn from the epicentre E to point 1 on the surface. For such rays the epicentral distance Δ (in degrees) and the time of transmission t (in seconds) are given by:

$$\frac{\Delta}{2} = 90 - i_0 \qquad (1)$$

$$\frac{t}{2} = \frac{1}{10}\, r_0 \cos i_0$$

i_0 being the angle of incidence.

The angle of incidence $\overline{i_0}$ of the ray which meets the core at grazing incidence is given by

$$\sin \overline{i_0} = \frac{5}{9} \qquad \overline{i_0} = 33^{o}.75 \tag{2}$$

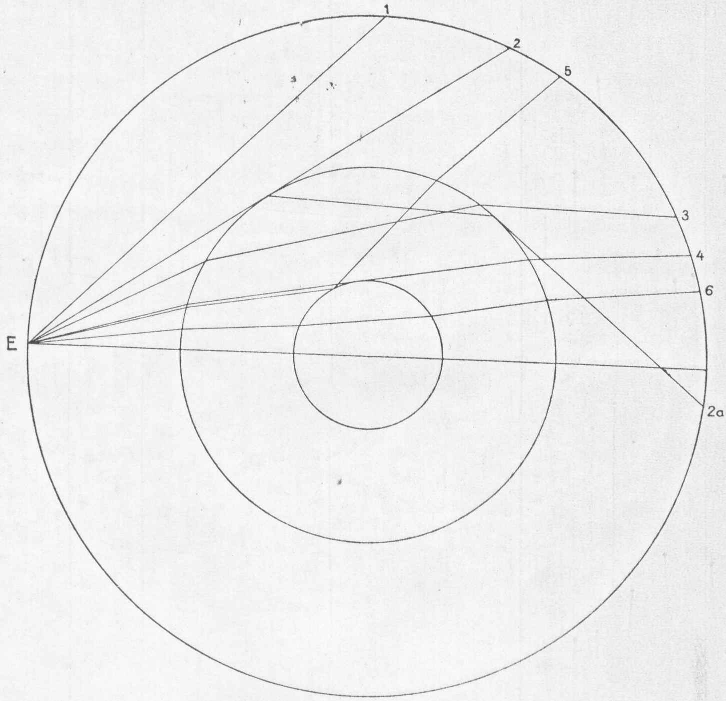

Fig. 1. — Paths through the earth.

The corresponding rays E 2 and E 2a have been drawn in the figure as well as the rays E 3 and E 4, penetrating the core.

For the rays penetrating the core we have :

$$\frac{\Delta}{2} = i_1 - i_0 + 90 - i_2$$

$$\frac{t}{2} = \frac{1}{10} r_0 \left(\cos i_0 - \frac{5}{9} \cos i_1\right) + \frac{1}{8}\cdot\frac{5}{9} r_0 \cos i_2 \tag{3}$$

i_1 and i_2 being angles of incidence at the outside and the inside of the core respectively and therefore determined by :

$$\frac{\sin i_1}{\sin i_0} = \frac{9}{5}; \quad \frac{\sin i_2}{\sin i_1} = \frac{8}{10} \qquad (4)$$

TABLE I

P and P'
calculated on assumption 1

i_0 Degrees	Δ Degrees	t m	s
45	90	15	1
44	92		17
43	94		32
42	96		47
41	98	16	2
40	100		16
39	102		30
38	104		44
37	106		57
36	108	17	11
35	110		24
34	112		36
33.75	112.5		39
33.75	186.2	26	30
33.65	178.6	25	43
33	167.9	24	38
32	161.6	24	0
31	158.2	23	40
30	156.2		29
29	155.0		22
28	154.3		18
27	154.0		17
26.5	153.9		16
26.25	153.9		16
26	153.9		16
25	154.1		17
24	154.4		19
23	154.9		21
22	155.5		24
21	156.2		26
20	157.0		29
19	157.8		32
18	158.7		36
17	159.7		39
16	160.7		42
15	161.8		45
14	163.0		48

i_0 Degrees	Δ Degrees	t m	s
13	164.0		51
12	165.1		54
11	166.3		57
10	167.5		59
9	168.7	24	1
8	169.9		3
7	171.2		5
6	172.4		7
5	173.6		8
4	174.9		9
3	176.2		10
2	177.6		11
1	178.8		11
0	180.0	24	11

Table I contains values of Δ and t calculated by (1) and (3) for various values of i_0. In Fig. 2, t as calculated from (3) has been plotted against Δ and the time-curve has been drawn. The points that correspond to integral values of i_0, measured in degrees, have been marked by open circles ; other points have been marked by crosses.

The P curve (see Fig. 4, p. 99) breaks off at $\Delta = 112^o.5$, corresponding to $\bar{i}_0 = 33^o.75$. For this angle of incidence we have also $\Delta = 186^o.2$ and the first point of the P'_2 curve.

For i_0 decreasing from $33^o.75$ to about 26^o the P'_2 curve is described in the direction indicated by the arrow ; it ends at an epicentral distance of $153^o.9$. When i_0 decreases from 26^o, the P'_1 curve, joining the P'_2 curve at a cusp, is generated ; it runs forward to $\Delta = 180^o$.

2. The energy contained in a small bundle of rays is proportional to the surface element

$$k \sin i_0 \, di_0 \, d\psi \tag{5}$$

on a sphere centred at the focus, i_0 being, as before, the angle of incidence and ψ the angle between the plane of the ray and an arbitrary fixed plane through the radius of the earth. Where the bundle of rays again meets the surface of the earth it cuts out the surface element

$$r_0^2 \sin\Delta \, d\Delta \, d\psi$$

and its cross-section is therefore

$$r_0^2 \sin\Delta \cos i_0 \, d\Delta \, d\psi \tag{6}$$

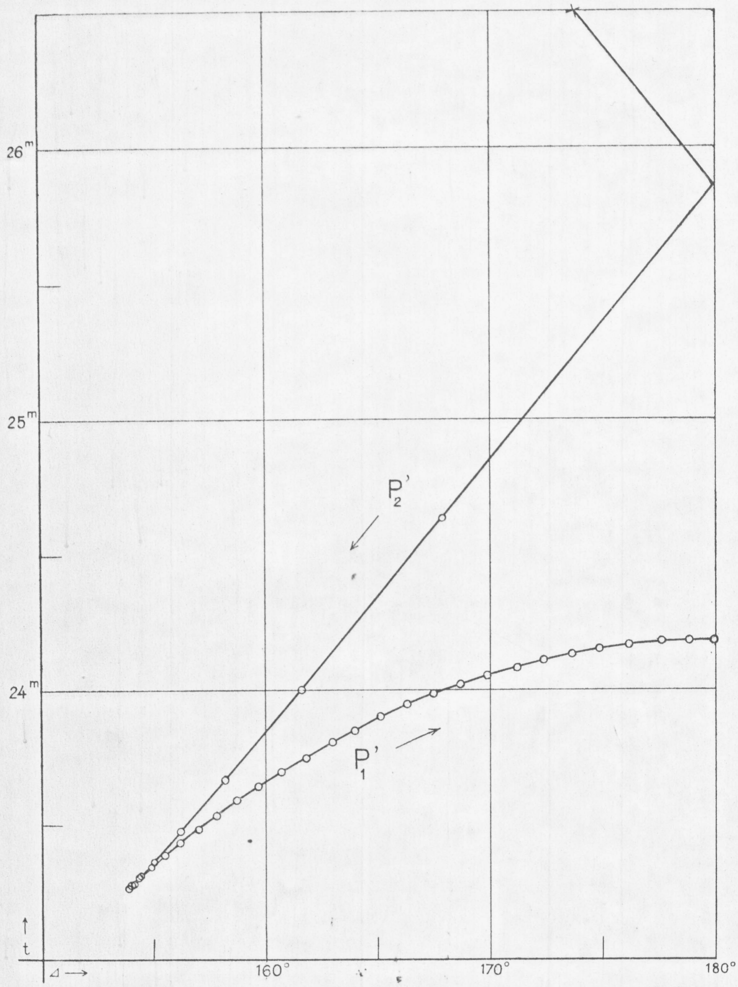

Fig. 2. — P' time-curves.

The intensity I of the ray is obtained by dividing (6) into (5) :

$$I = K \frac{1}{\sin\Delta} \tan i_0 \frac{di_0}{d\Delta} \tag{7}$$

It is hereby assumed that no energy is lost on the way whereas in reality energy is lost, especially at discontinuity surfaces, where part of the energy is reflected and part of it possibly refracted into a wave of different kind. For our purpose, however, the simple expression (7) is all that is needed.

3. From Table I divided differences of Δ with respect to i_0 may be computed. The differences of values of Δ corresponding to successive integral values of i_0 are approximate values of $\frac{d\Delta}{di_0}$ for the mid-points of the intervals. In fig. 2, therefore, $\frac{d\Delta}{di_0}$ is small where the crosses are close and large where they are far apart.

For the P'_1 curve $\frac{1}{\sin\Delta}\tan i_0$ does not differ greatly from 1, so that, according to (7), I is approximately proportional to $\frac{di_0}{d\Delta}$.

For the uppermost point of the P'_2 curve $\frac{di_0}{d\Delta} = 0$, as may be deduced from (3) and (4), and for the first part of the curve $\frac{di_0}{d\Delta}$ is quite small as illustrated by the lack of crosses. The energy of the corresponding ray, however, is not necessarily insignificant, for $\sin\Delta$ is quite small while $\tan i_0$ assumes greater values. This simply relates to the fact that energy originally sent out in different directions concentrates on a small area.

Conclusions as to the energy of P'_2 therefore cannot be drawn without further examination; but close to the cusp at which P'_2 turns into P'_1 the energy is great, for here the crosses are close together, indicating large values of $\frac{d\Delta}{di_0}$, and $\frac{1}{\sin\Delta}\tan i_0$ is greater than 1. There is thus a focal zone in which the P' amplitudes are very large; P'_1 and P'_2 both occur and the energy of both is great. They are quite close together, so close that it might not be possible to separate their pulses on a record.

4. The velocity distribution here assumed is simpler than the one actually found in the earth. The P' curves therefore deviate from those empirically determined ; their slopes and the critical distances are different. But, on the simpler assumption, both the P'_2 and P'_1 phases occur, and there is a focal zone where the energy of the ray is great. So far there is agreement with actual observations.

5. There are however observations which are not immediately explained on the assumption made here, nor do they find any explanation if gradually increasing velocity is assumed instead of constant velocity, or if the discontinuity surfaces in the upper part of the mantle are considered. It is actually found that the P curve does not break off abruptly, but continues in a curve corresponding to a ray of small intensity ; it is also found that P' is not confined to distances greater than the focal distance of about 143°, but is observed, though with smaller amplitudes, at smaller distances down to about 105°.

Diffraction has been resorted to for want of an explanation of the small P beyond about 100° as well as the small P' at distances smaller than the focal distance. At the discontinuity surface which separates mantle and core the entire energy would not be carried by refracted and reflected waves, but part of it would be spread in other waves carrying but a small amount of energy to the surface.

6. The P waves observed at distances greater than about 100° are always quite small and only recorded by the most sensitive instruments so that for a long time it was believed that they were not present at all.

Diffraction may afford an acceptable explanation of the occurence of these waves, but it is not the only possible explanation.

The epicentral distance Δ is expressed by :

$$\frac{1}{2}\Delta = \int_{r(\alpha)}^{r_0} \frac{\alpha}{r} \frac{dr}{\sqrt{u^2 - \alpha^2}} \tag{8}$$

where r is the distance from the centre of the earth and $u = \frac{r}{\varphi}$, φ being the velocity ; $\alpha = \frac{r_0}{\varphi_0}\, sin\, i_0$ is the constant of the ray, the index o being applied to surface values. $r\,(\alpha)$ is the value that r assumes at the deepest point of the ray. It satisfies the equation

$$\frac{r}{\varphi} = u = \alpha \tag{9}$$

and it is determined uniquely provided $\frac{r}{\varphi}$ is a monotonic function of r which decreases with decreasing r.

The integral in (8) converges if $\frac{du}{dr} \neq 0$. It may diverge if, at some value $r\,(\alpha)$ of r, $\frac{du}{dr} = 0$, and may assume very large values if $\frac{du}{dr}$ is quite small. It follows that Δ, considered as a function of α, may increase very strongly with α if $\frac{d\alpha}{dr(\alpha)}$ is quite small. Since $\alpha = \frac{dt}{d\Delta}$, $\frac{d^2t}{d\Delta^2} = \frac{d\alpha}{d\Delta}$ becomes quite small, i. e. the time-curve is practically a straight line, and the intensity of the ray, being proportional to $\frac{di_0}{d\Delta}$, is small.

The occurrence of small P beyond 100^0 may therefore be explained by the assumption of a layer of gradually decreasing velocity. The decrease must not be so strong as to make

$\frac{r}{\varphi}$ increase, but if $\frac{r}{\varphi}$ is very nearly constant, though still decreasing with decreasing r, the energy of the wave will become quite small and the time-curve will be practically a straight line.

Below the layer of gradually decreasing velocity a sudden decrease may be assumed. It is not necessitated by the occurrence of the P'_1 and P'_2 waves, but is required for the explanation of other waves.

7. The view has previously been held that the P' waves reaching the surface at distances smaller than the focal distance (hereafter, for simplicity, to be denoted P'_3) were small and insignificant. This has been found not to be so. The horizontal component of the movement is indeed small, but the vertical component may be considerable, and evidently the lack of vertical component instruments has caused the false impression to be gained of P'_3 being quite small.

In a study of the P' records of the Buller earthquake of 1929 June 16, I have drawn attention to the fact that P'_3 might be a strong wave and that the amplitude increased with increasing distance (6, p. 410). In Fig. 3 some P'_3 records of the Buller earthquake have been reproduced.

Later, other observers have also found that P'_3 might be of considerable amplitude. In a paper dealing with another New Zealand earthquake, the Hawke Bay earthquake of 1931 Febr. 2, C. G. Dahm (2) has reproduced the Georgetown Galitzin vertical component record from an epicentral distance of 124°.4. It is very much like the Tachkent record of the Buller earthquake at approximately the same epicentral distance.

B. Gutenberg and C. F. Richter also find strong vertical component records of P'_3 (see 3, Fig. 6, p. 89), especially on the Benioff short-period vertical seismometer, and as a consequence doubt that P'_3 is actually a diffracted wave.

J. B. Macelwane (7) and G. Krumbach (5) have both stud-

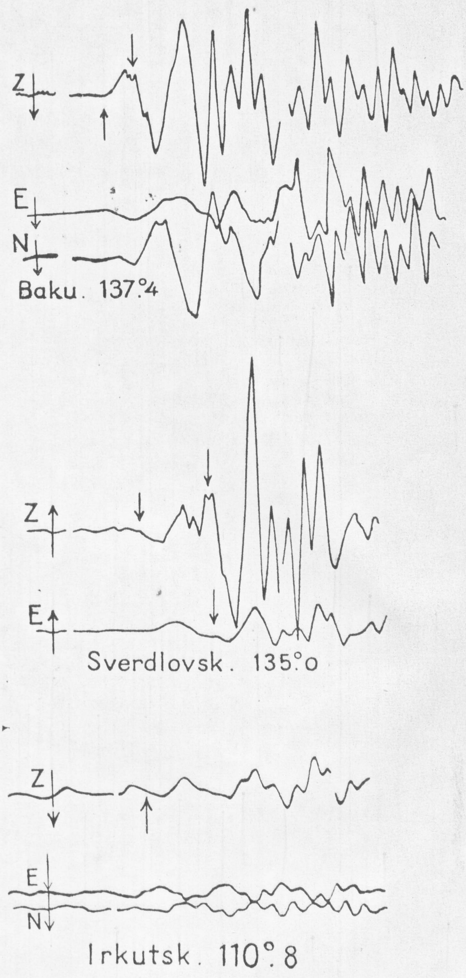

Fig. 3. — 1929, June 16. P'_3 records.

ied the Pacific earthquake of 1924 June 26 in which P' is well recorded, but there are no P'_3 vertical component records.

8. An explanation of the P'_3 wave is required, since now it can hardly be considered probable that it is due to diffraction. A hypothesis will here be suggested which seems to hold some probability, although it cannot be proved from the data at hand.

We take it that, as before, the earth consists of a core and a mantle, but that inside the core there is an inner core in which the velocity is larger than in the outer one. The radius of the inner core is taken to be $r_1 = \frac{8}{10} r_0 \sin 16^o = 0.2205\, r_0$, so that the ray whose angle of incidence at the surface of the earth is 16^o just touches the inner core. Taking the velocity in it to be 8.6 km/sec, the outer and inner angles of incidence i_3 and i_4 at its boundary are determined by

$$\frac{\sin i_3}{\sin i_2} = \frac{5}{9} \frac{r_0}{r_1}; \quad \frac{\sin i_4}{\sin i_3} = \frac{8.6}{8.0}. \tag{10}$$

i_4 does not exist for $i_0 > 14.86^o$. When $16^o > i_0 > 14^o.86$, the rays are totally reflected at the boundary of the inner core. Δ and t of the totally reflected rays are given by :

$$\frac{\Delta}{2} = i_1 - i_0 + i_3 - i_2 \tag{11}$$

$$\frac{t}{2} = \frac{1}{10} r_0 \left(\cos i_0 - \frac{5}{9} \cos i_1\right) + \frac{1}{8} r_0 \left(\frac{5}{9} \cos i_2 - \frac{8}{10} \sin 16^o \cos i_3\right)$$

and for the rays which pass through the inner core Δ and t are determined by :

$$\frac{\Delta}{2} = i_1 - i_0 + i_3 - i_2 + 90 - i_4 \tag{12}$$

$$\frac{t}{2} = \frac{1}{10} r_0 \left(\cos i_0 - \frac{5}{9} \cos i_1\right) + \frac{1}{8} r_0 \left(\frac{5}{9} \cos i_2 - \frac{8}{10} \sin 16^o \cos i_3\right) + \frac{1}{8.6} \cdot \frac{8}{10} \sin 16^o\, r_0 \cos i_4$$

In Table II values of Δ and t are given for values of i_0 from 16^o downwards.

TABLE II

P'
calculated on assumption 2

i_0 Degrees	Δ Degrees	t m	s
16	160.7	23	42
15	121.5	21	44
14.85	118.8	21	37
14	144.3	22	48
13	150.7	23	5
12	154.7		15
11	157.7		21
10	160.3		27
9	162.6		31
8	164.8		34
7	166.9		37
6	168.8		40
5	170.8		42
4	172.6		44
3	174.5		45
2	176.4		46
1	178.2		46
0	180.0	23	46

In Fig. 4, t has been plotted against Δ; for $i_0 \geqslant 16^o$ the values of Table I have been taken, for $i_0 < 16^o$ those of Table II. As in Fig. 2, the points which correspond to integral values of i_0 have been marked by open circles, other points by crosses.

9. The P'_2 curve in Fig. 4 is like that in Fig. 2. As before it joins a P'_1 curve at a cusp. But this curve breaks off at $\Delta = 160^o.7$ corresponding to $i_0 = 16^o$. The retrograde dotted curve starting at its end-point is the time-curve of the totally reflected wave; for $i_0 = 14^o.85$ it ends at $\Delta = 118^o.8$. At this point it is joined to the time-curve of the ray which passes through the inner core; for values of i_0 decreasing from $14^o.85$ to 0^o it runs forwards to $\Delta = 180^o$. It passes close to the endpoint of the P'_2 curve.

The retrograde dotted curve corresponds to a ray of quite small intensity, the difference between the angles of inci-

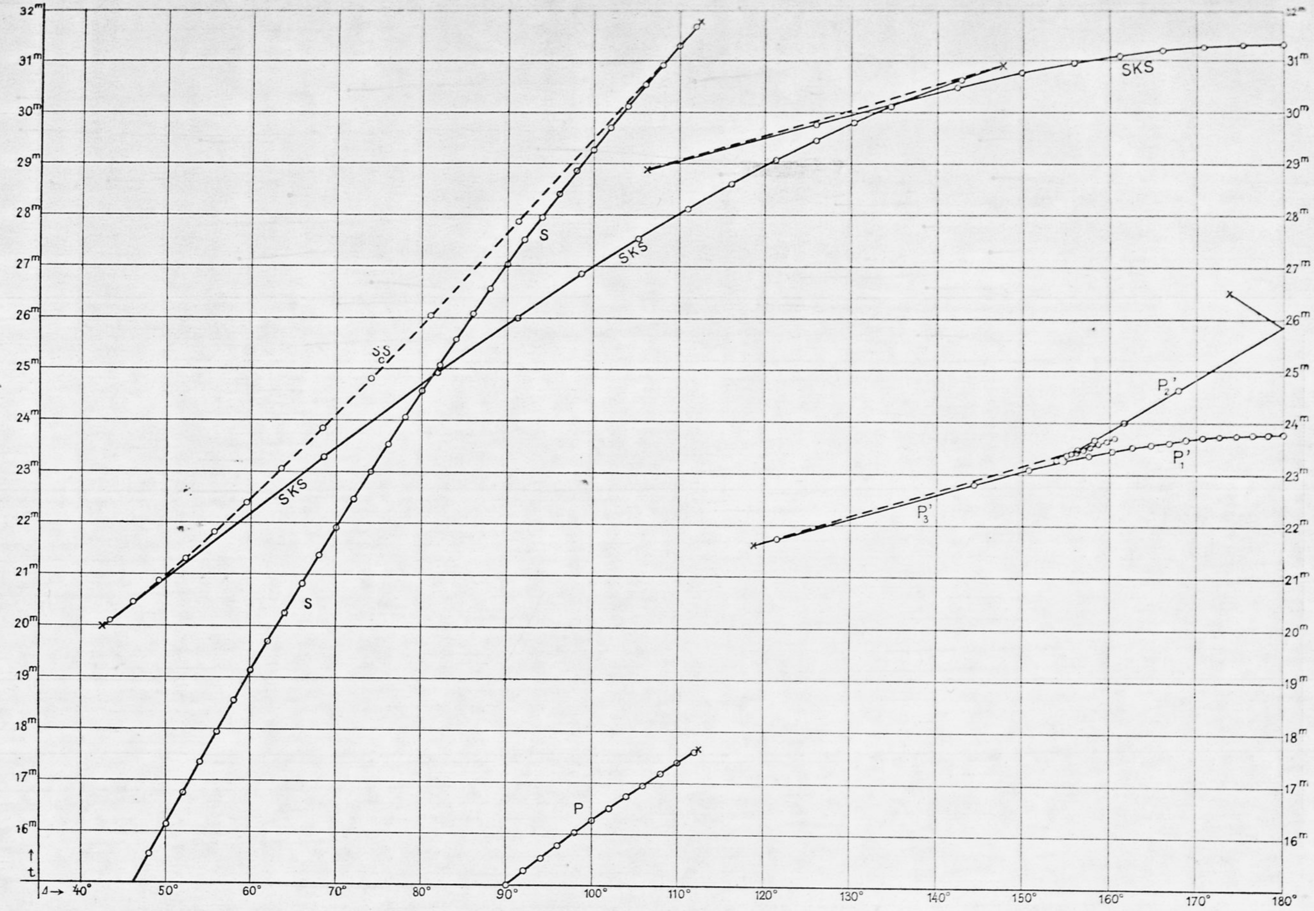

Fig. 4. — Time-curves.

dence at its starting point and at its end-point being only 1°.14.

The branch which joins on to it is the P'_3 curve. At the lower part of the curve, i_0 varies slowly with Δ so that the intensity of the ray is small. But the intensity of the P'_3 ray increases with increasing distance. For $i_0 = 13°$, we have $\Delta = 150°.7$ and for $i_0 = 12°$, $\Delta = 154°.6$. It is thus seen that, even at distances smaller than the focal distance, $\frac{di_0}{d\Delta}$ is no longer small; $\frac{\tan i_0}{\sin \Delta}$ being about $\frac{1}{2}$, the intensity of the ray is not quite small. The angle of incidence being small, the vertical component of the movement is relatively large.

These characteristics of P'_3 are the same as those actually observed.

10. The curve corresponding to the one usually termed the P'_1 curve forms in this case the continuation of the P'_3 curve. Joining on to P'_2 there is a further P'_1 curve.

C. G. Dahm (2) comes to the conclusion that the P'_3 and P'_1 curves are actually segments of one and the same curve. His data, however, are not convincing since observations are lacking in the most important interval of distance, that which includes the focal point.

B. Gutenberg and C. F. Richter (3) on the other hand are convinced that P'_3 stops at the focal distance; but their data are heterogeneous and there seems to be a possibility of changing the course of their P'_3 curves somewhat (see 3, fig. 5, p. 87).

In fig. 5 the fully drawn lines are the P' curve constructed from Jeffreys and Bullen's final tables (4, pp. 92-93). By means of the dotted lines they are transformed into curves of the type of the P' curves of fig. 4. The alteration is small and probably permissible on the data.

The fact that P'_3 and P'_1 both have large vertical components, i. e. small angles of incidence, is in favour of the supposition that they are due to one and the same wave. Also, according to my experience, there are in P'_3 oscillations of rather short period, the general appearance being much the same as that of P'_1.

11. The $P'_3P'_1$ curve of Fig. 4 passes close to the $P'_2P'_1$ cusp ; but a small change in the velocities or in the radius of the inner core is all that is required to make the $P'_3P'_1$ curve pass either above or below the cusp. In the figure of Gutenberg and Richter just referred to, there are several low points

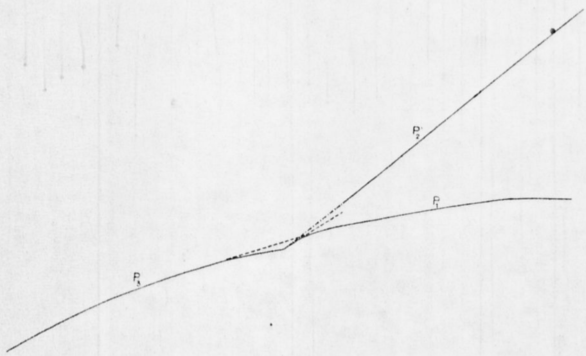

Fig. 5. — J.-B. P' curves with suggested alterations.

near the focal distance ; the cusp, therefore, seems to go below the $P'_3P'_1$ curve. This may also be inferred from fig. 5.

Theoretically it is possible that a cusp is not formed at all. If in 8 the limiting angle of incidence is taken $\geqslant 26^o$ instead of $= 16^o$, the time-curve of the wave reflected at the boundary of the inner core will form the backward continuation of the P'_2 curve. If the angle is not much greater than 26^o, there will still be a focal zone.

We have taken the increase of velocity to occur abruptly at the boundary of the inner core, but time-curves similar to those here considered may result if, instead of a discontinuous increase, a gradual, but very strong increase of velocity is assumed.

12. In Tables III and IV are tabulated the transmission times of S, ScS and SKS. The ratio of the velocities of S and P is taken to be 1.8, and for Table III the P velocities have been assumed to be as in 1, for Table IV those of 8 have been taken. In fig. 4 the times have been plotted against distance. For $i > 8^{\circ}.81$ the times of Table III have been taken, for smaller values of i those of Table IV.

TABLE III

S, *ScS* and *SKS*
calculated on assumption 1

i_0 Degrees	Δ Degrees	t m	t s
45	90	27	2
44	92		30
43	94		57
42	96	28	24
41	98		51
40	100	29	16
39	102		42
38	104	30	7
37	106		31
36	108		55
35	110	31	18
34	112		41
33.75	112.5	31	46
33	91.2	27	51
32	81.0	26	2
31	74.0	24	48
30	68.3	23	50
29	63.5	23	3
28	59.4	22	23
27	55.6	21	49
26	52.2	21	18
25	49.1	20	51
24	46.1	20	27
23	43.4	20	5
22.69	42.6	19	59
22	68.4	23	17
21	81.8	24	55
20	91.1	26	0
19	98.6	26	51
18	105.2	27	32

i_0 Degrees	Δ Degrees	t m	s
17	111.0	28	7
16	116.1	28	37
15	121.3	29	4
14	126.0		27
13	130.4		48
12	134.7	30	7
11	138.9		24
10	142.9		38
9	146.8		51
8	150.7	31	3
7	154.5		13
6	158.2		21
5	161.9		28
4	165.6		34
3	169.2		38
2	172.8		41
1	176.4		43
0	180.0	31	44

TABLE IV

SKS

calculated on assumption 2

i_0 Degrees	Δ Degrees	t m	s
8.81	147.6	30	54
8.19	106.2	28	52
8	126.0	29	46
7	142.3	30	29
6	149.9		46
5	155.9		58
4	161.1	31	6
3	166.1		12
2	170.8		16
1	175.3		18
0	180.0	31	19

The normal *S* curve ends at 112°.5. The dotted curve joining it is the *ScS* curve ; it runs backwards to 42°.6. At its upper part the intensity of the ray is quite small ; it is greatest at the smallest distances. The *ScS* curve turns into the *SKS* curve. The intensity of the *SKS* ray is at first small ; it increases with increasing distance. It crosses the *S* curve at about 81°. Where the curves cross, the intensity of *SKS* is still rather small and smaller than that of *S*, so that, on a record, the *SKS* pulse would not be easily observable where it comes after *S* and would appear as a small pulse in front of the larger *S* pulse where it first precedes it. In actual observations this is so, but on this point the agreement of calculations with observations is not significant, the actual velocity distribution being different from the one here assumed.

If there is no inner core, the *SKS* curve continues a smooth curve with decreasing inclination up to 180°, but if an inner core of radius r_1 is assumed the *SKS* curve breaks off at 147°.6 for $i_0 = 8°.81$. For angles of incidence between 8°.81 and 8°.19 the *SKS* ray is totally reflected and the corresponding time-curve runs backwards as far as 106°.2. The intensity of the ray therefore, is quite small. The time-curve of the *SKS* ray, which penetrates the inner core runs forwards from 106°.2 to 180°. For the smallest distances the intensity is quite small ; it increases with distance, but it is still not very large where at about 138° the curve crosses the first *SKS* branch.

The inner core thus produces a loop on the *SKS* curve, stretching from about 106° to 148°. The rays corresponding to the branches of the loop are of small intensity and are close to the other branches of the *SKS* curve, so that the existence of the loop would not be easily recognisable. At great distances the *SKS* curve as a matter of fact is not determined from observations ; the course, therefore, might well be similar to the one here described.

It may be noted, that a large amount of the energy contained in the transverse waves sent out from the focus goes into the reflected *ScS* waves and that the energy remaining for *SKS* is spread over a very large interval of distance. *SKS* therefore cannot be expected to be a strong wave.

II

P' records of the Buller earthquake

1. It will hardly be possible to determine with much certainty the time-curves of the P' phases around the focal distance unless detailed studies are made of earthquakes recorded by a group of stations at this distance. The phases are close together and therefore are not easily separated except

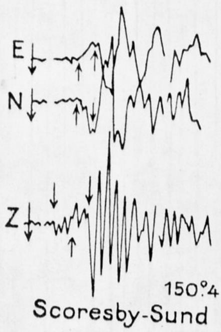

Fig. 6. — 1929, June 16. P' records.

by direct comparison of records of stations within a small range of distance.

Records of a group of stations in the focal zone have not been available, but the Buller (New Zealand) earthquake of 1929, June 16, previously studied (6), gave four P' records at distances around 150°. In the earlier investigation the first pulse only was considered in these four records, since it did not seem possible to interpret the later pulses. P'_2 was read at greater distances and was obviously present also at 150°, but it did not seem certain which of the later pulses were due to P'_2, and therefore no readings of this phase were given. The records have now been studied in detail and interpretation attempted.

The four stations are Ivigtut (149°.0), Abisco (149.9°), Scoresby-Sund (150.4°) and Pulkovo (150.6°). They are not in a group, but at great distances from each other. Their records, however, have common features.

The Scoresby-Sund P' records have been reproduced in Fig. 6. The beginning is quite small and is barely visible on the original Z record (not on the reproduction) ; there is an increase on Z where movement begins on N and E ; later there is a simultaneous increase of movement on all component records ; finally large oscillations set in on Z ; there is a simultaneous increase of movement on N and E, but the separation of phases is not very clear.

In the records of the other stations the movement increases in a similar way. The large oscillations are in all cases much larger than those of P'_1 or P'_2 at greater distances. At Scoresby-Sund, the magnification is much smaller than at other stations. At Pulkovo, with a more usual magnification, the recorded movement is very large and difficult to disentangle.

The following readings have been made :

TABLE V

Station	Δ	$P'Z$					$P'H$			
	Degrees	min.	sec.	sec.	sec.	sec.	min.	sec.	sec.	sec.
Ivigtut	149.0	7		! 12	! 20	29	7		! 20	
Abisco	149.9	7	7	13	22		7	12	22	
Scoresby-Sund	150.4	7	8	! 14	24	! 32	7	18	! 24	32
Pulkovo	150.6	7	9	! 17	! 26	34	7	! 17	26	32

The sign ! has been used to indicate distinct pulses ; where it has not been applied, the reading has been made at an

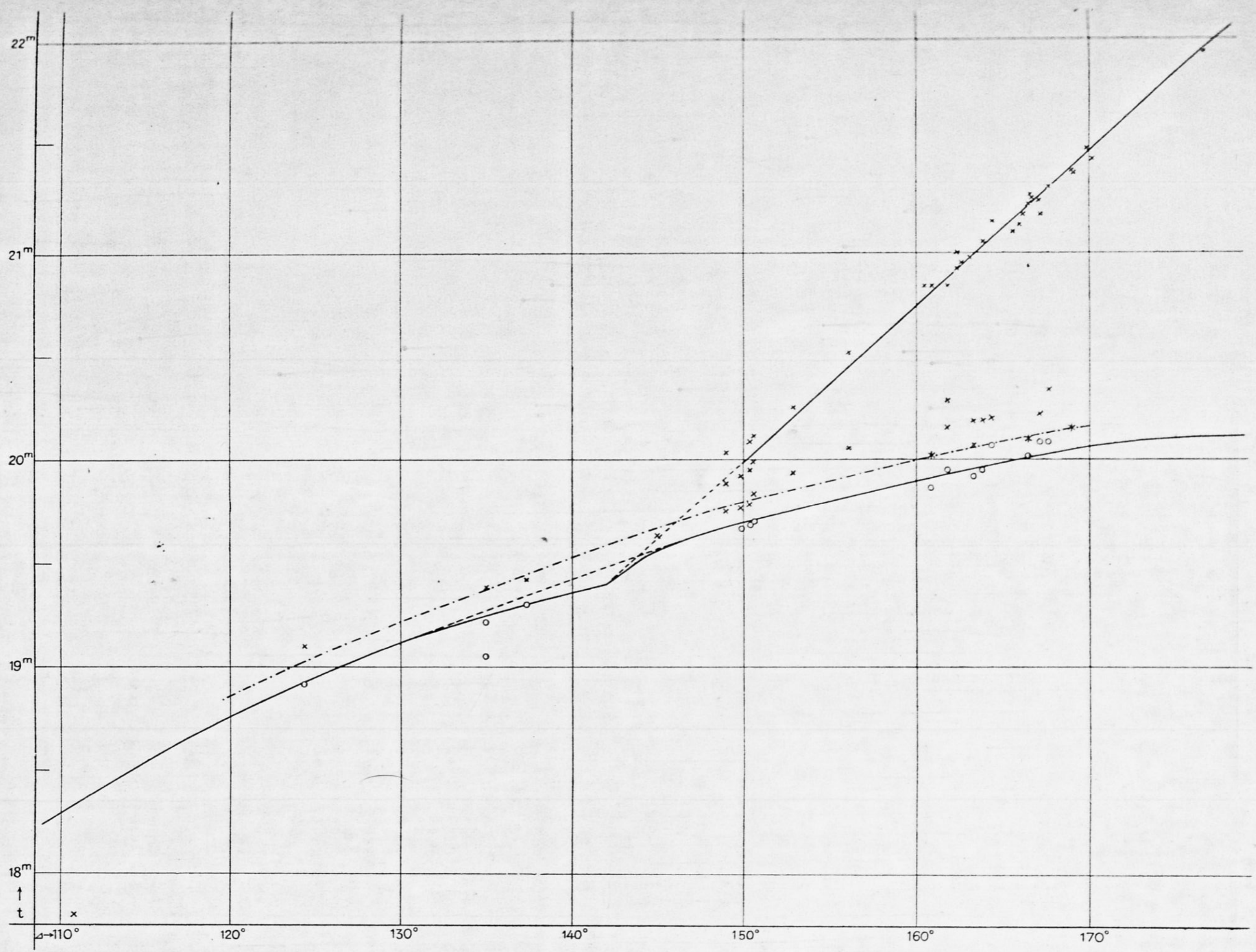

Fig. 7. — 1929, June 16. *P′* transmission times.

increase of movement but not at a definite pulse. For the Abisco readings however, the sign has not been used, since the records were not available for the final examination.

In Fig. 7 the corresponding transmission times have been plotted against epicentral distance together with other transmission times to be referred to later. Small and indistinct beginnings have been marked by open circles, other readings by crosses. $47^{m}27^{s}$ has been taken for T_0. The fully drawn curves are the P' curves of Jeffreys and Bullen's final tables (the J.-B. tables).

The points of the stations near 150^{o} are approximately on four lines. In the relatively weak Ivigtut Wiechert records, however, there is no pulse corresponding to the early faint beginning at the other stations and at Abisco there is no reading corresponding to the last pulse read in the other records ; but the Abisco records were difficult to read and the original reading was not done in such great detail as the later one for which they were not available.

The fact that the points near 150^{o} are on four lines may be taken as an indication of the existence of four distinct waves, even though some of the points are not due to very definite pulses in the records ; this applies more particularly to the points on the uppermost line.

2. If there is no discontinuity within the core we should at 150^{o} have barely two P' phases, P'_1 and P'_2, with a short interval between them, neither of them weaker than at greater distances. If there is an inner core the first phase P'_1 may be weaker than at greater distances. It should possibly be followed by a second, stronger P'_1 phase and there should, in any case, be a P'_2 phase, probably rather strong.

The observed construction of the P' phase at 150^{o} does not seem to be in accordance with either of these possibilities. There are four phases instead of 2 or 3.

3. The first phase however is quite weak. In the earlier

paper (6) it was menioned that the P' record of København as well as of other stations was introduced by quite a small vertical movement; the following movement had rather a strong vertical component and was also recorded by horizontal component instruments. A small introductory movement therefore is nothing special for distances around 150°.

This small first movement may have been caused by one or more small shocks immediately preceding the main shock. L. Bastings (1), from a study of the P records, arrives at the result that 3 shocks of different, but dynamically connected, origins have occurred. It does not, however, seem to me to be possible to make a clear distinction between P phases due to different shocks. At Wellington (2°.0) the first swing is small, but thereafter the trace vanishes. At distances of about 20°, small waves are recorded in front of large ones, but at this distance the risk is incurred of confusing P's due to different shocks with the P waves of one shock that arrive simultaneously owing to the existence of a discontinuity within the earth. The abnormally large transmission times found by Bastings rather indicate that such confusion is involved. At Adelaide (27°.0), at a distance where a single shock should not give rise to more than one P wave, there is first quite a small swing and then a somewhat larger one; finally, large oscillations set in. Bastings reads the onsets at: 10^s; 15^s; 22^s, whereas I read: 11^s; 23^s; 29^s.

At greater distances Bastings in several records reads 2 phases, at Batavia and at Manila 3, and the time intervals are approximately the same as at Adelaide. In the records I have seen, P as a rule is made up of increasing oscillations, but the separation of phases does not seem to me to be clear. At Batavia, Honululu and Manila the oscillatory movement is very strong throughout the records so that it does not seem certain that the oscillations read are due to different P's. At Manila there is a strong and marked increase of movement later than the pulses read by Bastings.

However, though the data are insufficient and a clear distinction between P's due to different shocks is not possible, there is little doubt that smaller shocks precede the main disturbance.

4. The small introductory movement observed in P' at distances between 160° and 170° as well as at about 150° is likely to be due to these smaller shocks. Since in my previous paper (6) the readings headed P'_1Z are readings of the first faint beginning of the phase, they are related to the foreshocks. These on the other hand are hardly recorded on the horizontal component instruments where the P'_1 movement is weak throughout, so that the readings headed P'_1H are likely to be due to the main shock. The corresponding points do not give a satisfactory determination of a curve, but this is not surprising since the weak and indefinite beginning of the phase cannot be equally well recorded by instruments of varying sensitivity.

An attempt was therefore made to determine the P'_1 curve of the main shock from the vertical component records. On all of them increase of movement takes place shortly after the faint beginning of the phase and the setting in of the stronger movement is likely to be due to the arrival of the P'_1 wave of the main shock. Readings were made of the onsets and tabulated in Table VI. When there is more than one reading there is an increase of movement at each. In fig. 7 the times are plotted against epicentral distance. They are marked by crosses. The open circles below are due to the P'_1Z readings of the earlier paper.

TABLE VI

Station	Distance	P'_1Z	
	Degrees	min.	sec.
Köbenhavn	160.9	7	19
			28
Wien	161.8		24
			36
			44
Hamburg	163.3		22
			31
			38
Jena	163.9		24
			38
Göttingen	164.4		31
			39
De Bilt	166.5		28
			33
Strasbourg	167.2		32
			40
Uccle	167.7		32
			47
Kew	169.0		36

The crosses give no better determination of a time-curve than do the circles. The cause of this may be instrumental, since even the stronger movement is weakly recorded at many stations ; but it is of course impossible to say whether other causes are not active.

The points of the only 3 Galitzin vertical component seismographs, those of Köbenhavn, De Bilt and Kew have been marked by double crosses ; the respective pulses are very clearly marked. The points are nearly on the line indicated in fig. 7, 6 sec above the J.-B. line. The line contains one further cross, but also a circle. The later increase of movement observed in some of the records is not very definitely due to new phases. On the Galitzin records the first half swing is not so large as the succeeding one but there is nothing to indicate that the increase of movement is due to the arrival of a new wave.

It seems therefore as if the line determined by the Galitzin points at distances between 160° and 170° may be taken to be the P'_1 curve of the main shock.

When the line is continued backwards, it is found to pass close to the second set of points at about 150°. The late points between 160° and 170° are about as high above the P'_1 line as is the third set of points near 150°, but, as said before, they hardly represent a distinct phase. Also, the strong records near 150° are hardly comparable with the records of the least sensitive instruments at greater distances. It seems likely, therefore, that the second set of phases read near 150° is P'_1 of the main shock.

These phases are, perhaps, more distinctly marked than any of the other P' phases read, but the movement in them is not very large. If the interpretation of the phase is correct, this fact points to the existence of an inner core.

5. The P' readings of the Russian stations at smaller distances are given below :

TABLE VII

Station	Distance	P'_3		
	Degrees	min.	sec.	sec.
Irkutsk	110.8	5	15	
Tachkent	124.4	6	! 22	! 33
Sverdlovsk	135.0	6	30	! 50
Baku	137.4	6	! 45	52

In fig. 7 they are plotted against epicentral distance. The early readings are due to the small pulses marked by the first arrows in fig. 3. Sverdlovsk has an exceptionally early point. The magnification of its instrument is very large and the movement recorded may be due to an earlier and weaker shock than the early P'_1 movement recorded elsewhere. The later points of Tachkent, Sverdlovsk and Baku are close to the backward continuation of the upper P'_1 line and are therefore probably due to P'_3 of the main shock.

The Irkutsk reading is very early, as are the P' readings of Tucson and Rio de Janeiro given in the previous paper. Other observers have also found very early P' at the smallest distances at which the phase has been read (see e. g. 4, p. 47). Since it is impossible to establish a connection between these early phases and P' as read at greater distances, the interpretation seems to be erroneous.

6. Either the third or the fourth set of pulses found in the records at distances of about 150° is likely to mark the arrival of P'_2 which should be stronger here than at greater distances.

The P'_2 readings of my earlier paper (6, p. 405) have been plotted in fig. 7 and the J.-B. P'_2 curve has been drawn.

Theoretically, the P'_2 curve is concave upwards ; however, in the P'_2 curve corresponding to Table I the concavity is slight. If the extreme points are joined by a straight line, no point of the curve departs more than 2 sec. from the line ; therefore a straight line can be drawn which will not depart more than 1 sec. from any point of the curve. If the curvature of the actual P'_2 curve is no greater, it is not observable, and the P'_2 curve can, in its entire length, be represented by a straight line.

The J. B. P'_2 curve is a straight line with the slope 4.4 sec /degree. My P'_2 points at distances between 160° and 170° seem to indicate a somewhat smaller slope and the slope I determined from them (6, p. 409) is 4.2 sec /degree.

The J.-B. P'_2 curve was constructed from Macelwane's readings of the 1924 June 26 shock combined with my readings here considered. Macelwane's P'_2 readings were found to be 6^s early as compared with mine, but then my early P'_1 readings were taken. We shall have to lower all my points 6 sec in order to make the P'_1 points of the main shock fall on the J.-B. P'_1 line ; then my P'_2 readings and Macelwane's will agree, but the J.-B. P'_2 line will be too high. This line cannot simply be lowered, for the backward production must meet the J.-B. $P'_3P'_1$ curve near its cusp. We shall have to turn the P'_2 line about the end-point of the backward production so much that between 160° and 170° the height is reduced by about 6 sec. Thus the slope is reduced by approximately 0.3 sec/degree to 4.1 sec/degree.

It is here assumed that my P'_2 readings are due to the main shock. They are in fact read where rather large movement sets in, and it would not be possible to trace the P'_2 of the small shocks in the movement following upon P'_1.

If in fig. 7 the slope of the P'_2 line is reduced to 4.1 or 4.2 sec/degree while the line is still made to pass « through » the points for distances between 160° and 170°, it will contain the fourth, not the third, set of points near 150°. The fourth set therefore should be due to P'_2.

The corresponding phases are less clearly indicated in the records than any of the other P' phases read; however, the movement following upon the third pulse is in all cases so very large and continues so long that it is more probably due to two waves than to only one. We cannot expect a great concentration of energy in either P'_1 or P'_2 at such great distance from the focal point.

Taking the uppermost points near 150° to be due to P'_2 we can interpret those below as the upper P'_1 phase corresponding to that of fig. 4.

7. We may summarise as follows: In four records at distances of about 150° pulses have been read which seem to indicate that there are four distinct P' waves at this distance. The first, small wave, recorded on vertical component instruments only, is likely to be due to small shocks preceding the main shock. The second wave which is not very large has been interpreted as the P'_1 wave of the main shock, recorded also at greater distances. At smaller distances the same wave is recorded as P'_3. The movement arising from the third and fourth waves is very large. The phases have

been interpreted as the upper P'_1 and the P'_2 phases. This interpretation presupposes the existence of an inner core.

It cannot be maintained that the interpretation here given is correct since the data are quite insufficient and complications arise from the fact that small shocks have occurred immediately before the main shock. However, the interpretation seems possible, and the assumption of the existence of an inner core is, at least, not contradicted by the observations; these are, perhaps, more easily explained on this assumption.

8. I hope that the suggestions here made may be considered by other investigators, and that suitable material may be found for studies of the P' curves. The question of the existence of the inner core cannot however be regarded solely from a seismological point of view, but must be considered also in its other geophysical aspects.

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